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A Theory of Extended Quantisation

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Abstract

A new theory of quantisation is presented. After arguments are given indicating that mass-energy in the universe is quantised, this quantisation is mathematically related to the lifespan and maximum size of the universe. Various consequences are then deduced, such as the existence of a minimum force.

The purpose of this paper is to motivate and to develop a theory of quantisation applying to all matter and energy. The theory very quickly leads to interesting results about the lifespan and maximum attainable size of the universe and dramatically shows the intimate relationship between the macrocosm and the microcosm. (In particular, both the lifespan and the maximum attainable size of the universe can be expressed in terms of the smallest possible unit of energy existing.)

One of the assumptions of this paper is that of a closed, expanding and contracting universe, finite and unbounded; it may or may not undergo only one oscillation. The universe is also assumed to be isotropic and homogeneous, with spacelike geodesics forming closed curves. The uncertainty principle is also assumed.

Notation: Throughout this paper $E(n)$ and $p(n)$ signify total energy and momentum, respectively, as functions of n. n is always a positive integer or zero.

1. *The Introductory Proposition; Arguments Supporting It*

The introductory proposition is that any portion of the mass-energy in the universe obeys the following quantisation principle:

> $E(n) = n\bar{k}$, where the quantum number n is a positive integer and $\bar{k}$ is the constant, basic, elemental unit of energy in the universe.

This proposition has already been rigorously derived for a universe *containing only radiation* and satisfying the assumptions in the second paragraph of this paper, in an article by Infeld & Schilds (1946). (Actually a much stronger result is derived by these authors, but using the independent method employed in the present paper, the stronger result—and other

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results as well—will be derived from this proposition.) It seems most reasonable to extend the proposition to a universe containing both matter and radiation.

There is also a heuristic argument supporting the introductory proposition. If a smallest possible unit of mass-energy $\bar{k}$, does not exist, then one is faced with the inescapable conclusion that mass-energy is infinitely divisible. However, the idea of infinite divisibility, in particular of mass-energy, has been found to be increasingly untenable. Since the last century, it has been discredited at several levels (the atomic structure of ordinary matter, quanta of radiation, nuclear particles, etc.), and is almost surely invalid at any level. Hence a smallest possible unit $\bar{k}$, applying to any mass-energy, should exist. And if it does, it seems probable that any mass-energy in the universe should occur in multiples of this unit; furthermore, it seems very likely that every multiple should (apart from selection rules for particular systems) at least be possible.

Finally, if the assumptions in the second paragraph of this paper are tenable, a result derived in Section 2 will indirectly but strongly support the proposition.

Now, suppose that the well-known relationship

$$m = \frac{m_0}{\sqrt{[1-(v^2/c^2)]}}$$

holds for a particle of rest-mass m_0 moving at velocity v. Then from $E = n\bar{k}$ it follows that

$$v = \sqrt{\left(1 - \frac{n_0^2}{n^2}\right)}c \qquad \text{and} \qquad p = \frac{1}{c}\sqrt{(n^2 - n_0^2)}\,\bar{k}$$

where $m_0 c^2 = n_0 \bar{k}$ and $mc^2 = n\bar{k}$.

2. *Relationships Between $\bar{k}$ and Parameters of the Universe*

In this section, x is the measured spacial displacement along a geodesic path (say that followed by an uninterrupted beam of light) which then can be regarded as a position coordinate.

Δ denotes 'uncertainty in the determination of', so that Δx denotes 'uncertainty in the determination of x', etc.

Now consider the uncertainty principle

$$(\Delta E)(\Delta t) \geqslant A\hbar$$
$$(\Delta p_x)(\Delta x) \geqslant B\hbar$$

where p_x is the momentum conjugate to the coordinate x, so that, optimally

$$(\Delta E)(\Delta t) = A\hbar$$
$$(\Delta p_x)(\Delta x) = B\hbar$$

where A and B are constants and $\hbar$ is, as usual, $h/2\pi$.

Now, for any measurement on any particle, regardless of its m_0,

$$E(n+1) - E(n) = \hbar \text{ so that } \{E(n+1) - E(n)\}_{\min} = \hbar$$

Thus, if the uncertainty in E, $\Delta E < \frac{1}{2}\hbar$, then $\Delta E = 0$. But if $\Delta E = 0$, Δt is infinite. This, however, is not possible. For what is $(\Delta t)_{\max}$? From the assumed theory of the universe, it may be the interval between one 'big bang' and the next; that is, the length of one cycle of the universe. However, this is finite. Thus $(\Delta E)_{\min} = \frac{1}{2}\hbar$. If the length of one cycle is denoted by τ, one then has from the uncertainty principle [since $(\Delta E)_{\min}$ and $(\Delta t)_{\max}$ or τ occur together]

$$\tau = \frac{2A\hbar}{\hbar}$$

Now assume $p_x = p$;

$$p(n+1) - p(n) = \frac{\hbar}{c}\sqrt{[(n+1)^2 - n_0{}^2]} - \frac{\hbar}{c}\sqrt{(n^2 - n_0{}^2)}$$

Regardless of m_0 (and thus n_0), it is easily seen that

$$\lim_{n\to\infty} [p(n+1) - p(n)] = \frac{\hbar}{c}$$

and that

$$p(n+1) - p(n) \geqslant \frac{\hbar}{c}$$

equality occurs here if $n_0 = 0$.

Thus $[p(n+1) - p(n)]_{\min}$ may be taken to be $\hbar/c$. Thus if $\Delta p < \frac{1}{2}(\hbar/c)$, then $\Delta p = 0$. But if $\Delta p = 0$, Δx is infinite. But again this is not possible. For $(\Delta x)_{\max}$ is the 'maximum circumference of the universe', this being the maximum uncertainty in the (geodesic) position coordinate measuring distance. This, from the assumed theory of the universe, is finite. Thus $(\Delta p)_{\min} = \frac{1}{2}(\hbar/c)$. If this 'maximum circumference' is denoted by U, one has [since $(\Delta p)_{\min}$ and $(\Delta x)_{\max}$ or U occur together]

$$U = \frac{2B\hbar c}{\hbar}$$

Now, if $A = B$,†

$$U = \tau c$$

† This supposition is, of course, questionable and is being made for simplicity only. Actually, at present, there is no way of being completely sure of the relationship between A and B. However, letting $2A = B$ gives $U = 2c\tau$, which agrees precisely with a result from a cosmological theory where one assumes a cycloidal universe with 0 cosmological constant. However, 'order of magnitude' agreement is sufficient here, and $A = B$ certainly gives that.

Suppose $B = \pi$†. Then

$$\not{k} = \frac{hc}{U} \qquad \text{or} \qquad \not{k} = \frac{h}{\tau}$$

Taking $\tau = 8{\cdot}2 \times 10^{10}$ years (the currently accepted value), it follows that

$$U = 7{\cdot}6 \times 10^{26}\ \text{m}$$

$$\not{k} = 2{\cdot}6 \times 10^{-52}\ \text{joule or } 2{\cdot}9 \times 10^{-69}\ \text{kg}$$

Independent derivations from ordinary cosmological theory support $U = c\tau$ to within a factor of 2.

The arguments of this section suggest that just as energy and momentum quantisation and a closed universe of finite duration go together, so an energy and momentum continuum and an infinite universe of infinite duration would go together.

3. *The Minimum Possible Force*

The previous sections imply a significant fact about force F usually defined as dp/dt. One must now write

$$F = \frac{\Delta p}{\Delta t}$$

where Δ now indicates 'the change in' rather than 'the uncertainty in'.

Now,

$$(\Delta p)_{\min} = \frac{\not{k}}{c}$$

exactly

$$(\Delta t)_{\max} = \tau$$

as defined in Section 2. Thus there actually may exist a minimum force which is given by

$$F_{\min} = \frac{\not{k}}{c\tau} \approx 3 \times 10^{-79}\ \text{newtons}$$

The existence of $F_{\min}$ is not academic; consider the gravitational force between 2 electrons at a geodesic distance of 3×10^4 m. Under suitable

† Originally, the author tentatively chose the value 1. However, after having read the completed paper, Dr. Bernard Luffman suggested that always $\lambda = (h/p) \leqslant U$. Following this suggestion, it is immediately seen why π is chosen. For $p_{\min} = (\not{k}/c)$, so that $\lambda_{\max} = hc/\not{k}$. Choosing π then gives $U = \lambda_{\max}$ exactly, a most plausible situation. If it is assumed that $2A = B$, then $A = \pi/2$, which gives agreement with equation (9.92) of Infeld & Schild (1946).

(and quite realisable) conditions, Newton's formula gives a fairly good (classically valid) approximation

$$F \approx 0{\cdot}7 \times 10^{-79} \text{ newtons}$$

which, however, is $<F_{\text{min}}$. Thus F equals 0 actually (in an operational sense).

It is widely suspected that the size of the universe depends on the quantity of mass-energy it contains. Thus it should be possible (eventually) to express this quantity in terms of U, τ, $\hbar$.

Reference

Infeld, L. and Schild, A. (1946). *Physical Review*, **70**, 410.